Optimal Selling Timing for Housing Developers

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Abstract

Deciding when to sell is an essential task for housing developers. In this paper, we have developed a theoretical function that can determine the optimal selling time for developers by estimating optimal home selling prices within a real option framework. The findings of our numerical analysis indicate that developers can delay pre-sales longer (make pre-sale agreements later) as initial investments grow. A longer project period has a converse effect on the optimal selling time. Developers who have the ability to obtain funding will delay pre-sales longer, pending additional market information that will benefit their corporate vision. Our results provide an optimal selling time strategy that can be used as a reference for developers.
Keywords: optimal selling time, real option, comparative static analysis

1. Introduction

It is widely realized that the decision to invest is a decision to exercise a real option and that many insights from the theory of financial options apply to real investment decisions (Dixit and Pindyck, 1994). It is same conception for housing selling timing issue. Selling timing, decisions for pre-sales or sale after completion, is an important issue for housing market. Pre-sale contracts allow developers to disperse the construction cost and housing price volatility in the future. In addition, housing buyer also can avoid for large sum of money at once and confirm housing price as pre-sale contract was established. On the other hand, housing sell later, even it had been completed, let developer hold exercise right longer for predominating more market information. Caballero (1991) argues that imperfect competition is vital to predicting a negative relationship between uncertainty and investment. However, previous researches of real estate have rarely been studied for optimal selling timing (or best selling timing) for housing with comprehensive theoretical model. Lai et al. (2004) model a pre-sale decision in a real options framework and some researches also provide empirical evidence that supports the “option to wait” theory (Quigg, 1993; Bulan et al., 2009; Schwartz and Torous, 2007; Buttimer et al., 2008). Housing pre-sale is a common phenomenon in many Asian real estate markets, it had become more attention in western countries in recent years. How to make the selling timing decision for developers? In this paper, we develop a real option model to realize the selling timing issue.

The ability to delay investment decisions is valuable when the investment is irreversible and the future is uncertain (Bulan, 2005). If managers can wait for the resolution of uncertainty before deciding to pursue the irreversible investment, they can avoid potentially large losses by foregoing the investment altogether when the outcome is unfavorable. Real option models apply most directly to individual investment projects and predict that trigger prices (Bulan et al., 2009). Studies that use project level data have been able to investigate the effect of uncertainty on the timing (as opposed to the level) of investment, which is a direct test of the optimal exercise of real options (Hurn and Wright, 1994; Quigg, 1993; Moel and Tufano, 2002; Collett et al., 2003).

Landowners have to decide what time to develop (e.g., for residential or commercial). Decisions of this type include land-redevelopment decisions where the density of residential or commercial development. Developer starts a construction expected a projected future demand. They take the risk that the demand might not be realized when the projects are completed. Williams (1991) indicates the owner optimally develops his property as development maximizes its market value if investors can trade substitute securities continuously without transaction costs in a perfectly competitive capital market. Developers will be in a difficult financial position if demand has unexpectedly fallen at the time of completion. Fortunately, pre-sale contracts provide solutions. Collett et al. (2003) indicate that high transaction costs are associated with longer holding periods and higher return volatility will cut short the holding periods which are provided by proportional hazards model. Lai et al. (2004) mentioned that the pre-sale method not only helps developers deal with the uncertainty of future demand and potential bankruptcy costs, but can also substantially reduce, even clear up, developers' inventory costs (holding vacant buildings). Wong et al. (2006) argue that futures market system reduces the volatility of spot prices in real estate market. Chan et al. (2008) observe that pre-sales mitigate a developer's financing constraint by providing development equity and reducing financing costs. Under the same concept of development timing, in this paper, considering the real options framework, we estimate the optimal housing price to sell then decide the decision of selling timing. Developers need to determine the housing selling timing, make pre-sale contracts or sell it as the construction has been completed.

The pre-sale contracts in housing market has been very popular in Asia, however, little attention has been paid to the selling timing characteristics of this strategy for developers. We make two significant contributions towards finding support for optimal selling timing of housing. First, developing a housing developer's selling strategy using a real options framework, considering multidimensional determinates for optimal housing price to sell and then realizing the optimal selling timing decision. Second, we pay attention on determinants which effect optimal housing selling timing by sensitivity analysis. Our model, in this paper, is established on developers perspective. Following the concepts of land development timing to develop a comprehensive theoretical model by real option framework. Compared with similar research issue or model, Capozza and Li (2002) models the development decision when net rents are growing geometrically and uncertainly, and capital intensity is variable. Lai et al. (2004) analyze the rationale for the existence of the presale method and develop strategies to design a presale contract using a real-options framework. Chan et al. (2008) provide a simple equilibrium model in a game-theoretical framework to throw light on the relationship between presales and developers' pricing and production decisions. Edelstein et al. (2012) provide a theoretical foundation on market equilibrium analyses to explain the extensive worldwide practice of residential real estate presale agreements for uncompleted dwellings.

We consider many characteristics of corporation to develop an integrated estimation, such as the housing price volatility, housing return, depreciate rate, capital structure of developer, risk-free rate, corporate tax rate, initial investment, initial housing price, related cost and the integrated period. Then, estimating optimal housing price to get imply optimal selling timing and providing suggestions of selling decisions for developers. Our application of real option to selling decisions provide a number of important new results by sensitivity analysis for the determinants of optimal housing selling timing. We can generalize that housing price volatility, initial investment, capital structure of developer, depreciation rate, corporate tax rate and risk-free rate are positive relationship with optimal selling timing. On the other hand, housing return, h**o**using price growth rate and integrated period are negative relation with optimal selling timing. Our model can provide the optimal selling timing strategy for developers by sensitivity analysis. Furthermore, we also can realize the implied meaning for selling timing.

Previous researches indicated that optimal timing issues had become a popular practice for property developments. Follow the same concept, we develop a real option model to describe housing selling situation. This chapter is organized as follows: Chapter 2 “Literature Review” reviews the literature as well as highlights the previous studies on the selling decisions of residential units. Chapter 3 “Real option model for optimal development timing” derives the optimal land development decision by real option framework. Chapter 4 “Sensitivity analysis of the best timing to develop” presents the comparative static results and strategies influence. Chapter 5 “Chapter Summary.

2. Literature Review

Theoretical work suggests that if the future is uncertain and an investment is durable and illiquid, then the ability to pursue a different investment (or not to invest at all) in the future has economic value, often referred to as a “real option” (McDonald and Siegel, 1985; McDonald and Siegel, 1986; Titman, 1985; Lai et al., 2004; Buttimer et al., 2008). Cunningham (2007) also considers the irreversibility condition for real options. Built structures satisfy these two conditions that are both highly durable and inseparable from land if without demolition. Reviewing optimal timing issues, most researches focus on the land development part, which considered the relationship between the volatility of land price and option premium.

Williams (1991) argues that real estate development is much like exercising an option. He models an option which can solve analytically and numerically for the optimal date and density of development, the optimal date of abandonment, and the resulting market values of the developed and undeveloped properties. Capozza and Helsley (1990) and Batabyal (1996) have addressed the question of land development under uncertainty in a many-period setting. Real options in land markets arise from uncertainty as to the optimal use of a site (Cunningham, 2007). Land may be undeveloped to preserve its real option value. Bulan et al. (2009) have evidence that increases in both idiosyncratic and systematic risk lead developers to delay new real estate investments. They find one-standard deviation increase in the return volatility reduces the probability of investment by 13 percent, equivalent to a 9 percent decline in real prices. It is now widely accepted that the decision to develop a building site is ultimately a choice to exercise a real option (Cunningham, 2007).

The land development question is modeled in a dynamic and stochastic framework. In this setting, an answer is provided to the when-to-develop-land question. This answer involved a comparison of the revenue obtainable from developing at time *t*, with the expected revenue to be obtained by preserving and waiting for new information beyond time *t*. To measure development timing, Capozza and Li (2002) model the development decision when net rents are growing geometrically and uncertainly, and capital intensity is variable. Collett et al. (2003) use a proportional hazards model to shed light on investor behavior with marked differences by type of property and over time. They mentioned that high transaction costs are associated with longer holding periods. Return volatility, by contrast, is associated with shorter holding periods. In addition, they also support that there might be an optimal holding period. Wong et al. (2006) examine the effect of forward sale (pre-sale) activities on the volatility of spot prices in the real estate market. The results contribute to the long lasting debate on whether the introduction of a futures market reduces the volatility of spot prices. Cunningham (2007) estimates proportional hazard models which includes time-invariant measures of building-site quality, time-varying measures of new-housing demand, and time- and location varying measures of expected future prices and price uncertainty. Numerous scholars have presented the land development with real option framework. However, previous researches rarely mentioned housing selling timing. Will developers make pre-sale contracts or sell it as completed? Any characteristics effect the selling decisions of developer? We follow the optimal development concepts to solve the selling timing issues for housing.

Developers consider to start a construction based on a projected future demand and take the risk that the demand might not be realized when the projects are completed. Lai et al. (2004) mentioned that there are at least three methods to deal with the risk associated with demand uncertainty. One of the methods is to presell a project. The pre-sale system offers developers an opportunity to share risks with buyers (Chan et al., 2008). Pre-sale agreements have become a pervasive worldwide practice for residential sales, especially in many Asian markets. Residential builders use pre-sale real estate agreements in order to secure buyers for either un-built or under-construction dwellings. Shyy (1992) and Lai et al. (2004) develop two prior theoretical models for pre-sales contracts, based upon compound real options. Chan et al. (2008) indicate that a developer can sell a property before its completion (or even before its construction) by pre-sale method.

There are many pre-sale cases have become a popular practice for property transactions in practice. However, we can find only a few studies to address pre-sale issue. One group of studies treats a pre-sale as a forward or futures contract (ignoring a buyer's option to default) and addresses issues such as pricing factors (Chang and Ward, 1993) and the relationship between pre-sale pricing and future spot sale pricing (Wong et al., 2006). Another stream of the literature treats a pre-sale contract as an option contract, and addresses issues such as its impacts on a developer's development strategies (Lai et al., 2004) and its influence on market structure (Wang et al., 2000). Bulan (2005) investigates real options behavior in capital budgeting decisions. Cunningham (2007) explores growth controls (specifically new development density restrictions) in the presence of the real options and empirically tests a theoretical prediction that growth controls may affect the timing of land development differentially in the presence of uncertainty. Chan et al. (2008) explores the impacts a pre-sale contract has on a developer's pricing and production decisions in a game theoretical framework. Edelstein et al. (2012) create a set of interrelated theoretical models for explaining how and why developers and buyers engage in pre-sale contracts for non-completed residential dwellings. Housing pre-sale is a common phenomenon. How to make the selling timing decision for developers? It is an important issue. We estimate the optimal housing price for the implied selling timing in response.

3. Real option model for optimal development timing

To model the effects of a presale contract in the real estate market, we begin with a real option framework which can make the optimal housing price decision and get implied development timing as maximizing developers' market value. Follow this model, we can realize the pre-sale developed timing for real estate. Is there pre-sale agreements or sell it as the project completed. Moreover, we can find out the influence for pre-sale developed timing from variables.

$$\frac{dH}{H} = \left(u-\delta\right)dt + \sigma dW \tag{3.1}$$

where

H is housing price.
u is housing price return.
δ is housing depreciate rate.
σ is standard deviation of housing price (housing price risk).
α is housing price growth rate.
$\alpha = u - \delta$

Developers have the choice for what time to sell the housing project before the project completed, therefore, it is a real option for developers. The developer has to be charged with related costs, *b*, for holding the land to develop (Markusen and Scheffman, 1978). The related costs such as the payment of land buying, design and construct the building, interest payment, etc.. The real option value will satisfy the partial differential equations within the risk neutral assumptions as follow:

$$F_t + 0.5F_{HH}H^2\sigma^2 + F_H(r-\delta)H - rF - b = 0 \tag{3.2}$$

where

b is related costs.

the general solution for equation (2) is

$$F\left(H\right) = A_0 + A_1H^{\beta_1} + A_2H^{\beta_2} \tag{3.3}$$

the boundary condition as follow:

$$F\left(0\right) = \frac{-b}{\alpha\text{-}u}\left(e^{(\alpha\text{-}u)T} - 1\right) \tag{3.4}$$

$$F\left(H^*\right) = H^* - I - \frac{b}{\alpha\text{-}u}\left(e^{(\alpha\text{-}u)t} - 1\right) \tag{3.5}$$

where

T is the integrated period.
H^* is optimal selling timing.

I is the investment from developer at t_0.

T is the periods from developer become the land owner to complete the construct project. We assume the invest payment have been paid as the beginning. t is the selling timing of the construct project from equation (3.4). As $t < T$ means it's a pre-sale agreement. We consider t is a function of optimal selling price of housing, H^*. In other words, H^* is the price at optimal development timing. The optimal selling price of housing equal to $H^* = H_0 \cdot e^{\alpha t}$. Equation (3.5) will be modified to equation (3.6).

$$F\left(H^*\right) = H^* - I - \frac{b}{\alpha\text{-}u} \times \left[\left(\frac{H^*}{H_0} \right)^{\frac{\alpha\text{-}u}{\alpha}} - 1 \right] \tag{3.6}$$

where

H_0 is the housing price at t_0.

Crowding to equation (3.4) and (3.6), the general solution of equation (3.2) will be

$$\begin{aligned} F\left(H\right) &= \frac{-b}{\alpha\text{-}u}\left(e^{(\alpha\text{-}u)T} - 1\right) + H^*\left(\frac{H}{H^*}\right)^{\beta_1} - I\left(\frac{H}{H^*}\right)^{\beta_1} \\ &- \frac{b}{\alpha\text{-}u}\left[\left(\frac{H^*}{H}\right)^{\frac{(\alpha\text{-}u)}{\alpha}} - 1\right]\left(\frac{H}{H^*}\right)^{\beta_1} + \frac{b}{\alpha\text{-}u}\left(e^{(\alpha\text{-}u)T} - 1\right)\left(\frac{H}{H^*}\right)^{\beta_1} \end{aligned} \tag{3.7}$$

Tax shield would be existing because of the developers build constructions with rising debt (Markusen and Scheffman, 1978; Anderson, 1986; Bulan, 2005):

$$\text{TS} = \frac{1}{r_b}\gamma \cdot I \cdot t_c \cdot \left(e^{r_b t} - 1\right) \tag{3.8}$$

where

γ is capital structure of developer.
t_c is the corporate tax rate.
r_b is the cost of debt

The benefit for developers would equal to the housing selling income deduct from related costs and investment at beginning. Furthermore, coupled with tax shield effect:

$$F\left(H^*\right)=\frac{-b}{\alpha\text{-}u}\left(e^{(\alpha\text{-}u)T}-1\right)+H^*\left(\frac{H}{H^*}\right)^{\beta_1}-I\left(\frac{H}{H^*}\right)^{\beta_1}$$

$$-\frac{b}{\alpha\text{-}u}\left[\left(\frac{H^*}{H}\right)^{\frac{(\alpha\text{-}u)}{\alpha}}-1\right]\left(\frac{H}{H^*}\right)^{\beta_1}+\frac{b}{\alpha\text{-}u}\left(e^{(\alpha\text{-}u)T}-1\right)\left(\frac{H}{H^*}\right)^{\beta_1}+\frac{1}{r_b}\gamma\cdot \mathrm{I}\cdot \mathrm{tc}\cdot\left(e^{r_b t}-1\right) \tag{3.9}$$

Then, we use the smooth-pasting condition approach with equation (3.9) to get the optimal selling price of housing:

$$H^*=I+\frac{b}{\alpha\text{-}u}\left(1-e^{(\alpha\text{-}u)T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1\alpha} \tag{3.10}$$

$$\beta_1=0.5-\frac{(r-\delta)}{\sigma^2}+\sqrt{\left[\frac{(r-\delta)}{\sigma^2}-0.5\right]^2+\frac{2r}{\sigma^2}} \tag{3.11}$$

where

r is risk-free rate.

There is a growth rate in real estate market. It is the best selling timing for developers as the housing price become H^*. We assume developers would rationally choice the optimal developed timing to make pre-sale agreements basing on maximize their market value. Developers would sell dwellings earlier with lower H^*. As $t<T$, which means developers make pre-sale contracts. On the other hand, Developers would sell dwellings later with higher H^*. As $t>T$, which means developers sell the dwellings as accomplished.

$$E(H_t)=H_0\times e^{\alpha t} \tag{3.12}$$

$$t^*=\frac{\ln\left(\left[I+\frac{b}{\alpha\text{-}u}\left(1-e^{(\alpha\text{-}u)T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1\alpha}\right]\times\frac{1}{H_0}\right)}{\alpha} \tag{3.13}$$

The land development question is modeled in a dynamic and stochastic framework. In this setting, an answer is provided to the when-to-develop-land question. This answer involved a comparison of the optimal developed timing from developing at time t^*. Equation (3.12) shows expected price of sold housing at time t. The optimal developed timing for pre-sale is showed at equation (3.13) with the expected price of sold housing equal to optimal housing price to sell. H_0 is the housing price at start date of planning. Developers would sell dwellings more earlier even the dwellings is not completed when t^* is lower, in other words, developers make pre-sale contracts. And vice versa.

4. Sensitivity analysis of the best timing to develop

We employ sensitivity analysis to investigate the effective to optimal selling timing from variables. Such as the housing price volatility, housing return, initial investment and capital structure of developer. We develop a general function with real option framework to realize the best timing to sell. For the purpose of the sensitivity analysis, the real estate market parameters, the interest rate and the taxes should be obtained. We summarize the parameters and numerical values for the real estate market and mortgage parameters in Table 4.1.

Table 4.1: Real estate market parameters

Real estate market parameters	Symbol	Numerical values
Housing return	u	20%
Housing depreciate rate	δ	3%
Housing price volatility	σ	20%
Capital structure of developer	γ □	60%
Risk-free rate	r	1%
The cost of debt	r_b	3%
The corporate tax rate	t_c	20%
The investment from developer at t_0	I	30,000,000
The housing price at t_0	H_0	30,000,000
Related costs	b	1,000,000
The integrated period (year)	T	2

4.1 Housing price volatility

$$\frac{\partial \beta_1}{\partial \sigma}=\frac{2}{\sigma^3}\cdot\left\{(r-\delta)-\frac{\left[\frac{(r-\delta)}{\sigma^2}-0.5\right]\cdot(r-\delta)+r}{\sqrt{\left[\frac{(r-\delta)}{\sigma^2}-0.5\right]^2+\frac{2r}{\sigma^2}}}\right\} \tag{4.1}$$

$$\frac{\partial t^*}{\partial \beta_1}=\frac{-(I\cdot\gamma\cdot t_c-b)}{\left[I-\frac{b}{\delta}\left(1-e^{-\delta T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1(u-\delta)}\right]\cdot\left[\beta_1(u-\delta)\right]^2} \tag{4.2}$$

Housing price volatility is a function of β_1 also as equation (3.4.1) showing. The relationship between risk-free rate and β_1 would effect positive/negative sign between optimal developed timing and β_1 in equation (3.4.2). We find out the relationship between t^* and housing price volatility by numerical approach with our parameters setting which shows in Figure 4.1. We can realize the positive trend for t^* as sigma increasing, moreover, the marginal increase become lower as sigma getting higher. The positive relationship also indicates developers would prefer to sell housing when it completed as housing price volatility increased. Same way, developers are more motivated to sell housing later, even completed, because the housing price maybe get a higher level with higher housing price volatility (Titman, 1985; Williams, 1991; Cunningham, 2007).

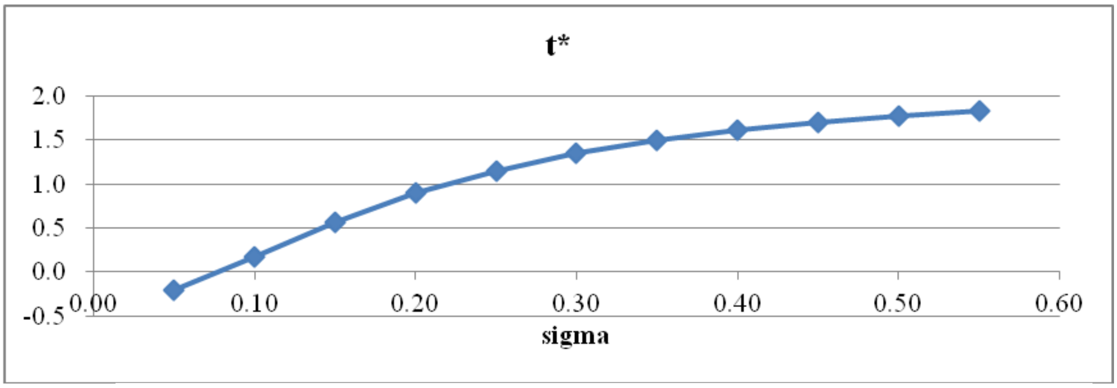

Figure 4.1. The optimal selling timing is changed by housing price volatility

4.2 Housing price growth rate

$$\frac{\partial t^*}{\partial \alpha}=\frac{\left\{\frac{\left[\frac{-b\cdot\left[1+\left[(\alpha-u)\cdot T-1\right]\cdot e^{-\delta T}\right]}{(\alpha-u)^2}+\frac{\beta_1\cdot(\gamma\cdot I\cdot t_c-b)}{(\beta_1\alpha)^2}\right]}{\left[I+\frac{b}{(\alpha-u)}\left(1-e^{(\alpha-u)T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1\alpha}\right]}\right\}\cdot\alpha+\ln\left\{\left[I+\frac{b}{(\alpha-u)}\left(1-e^{(\alpha-u)T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1\alpha}\right]\cdot\frac{1}{H_0}\right\}}{\alpha^2} \qquad (4.3)$$

The relationship between housing growth rate and optimal developed timing shows on equation (4.3). Figure 4.2 shows the numerical result and indicates negative relationship trend. The result determines developers would prefer to launch pre-sale contracts as housing price growth rate increasing. Developers would not to hold the right of selling timing as housing growth rate is higher.

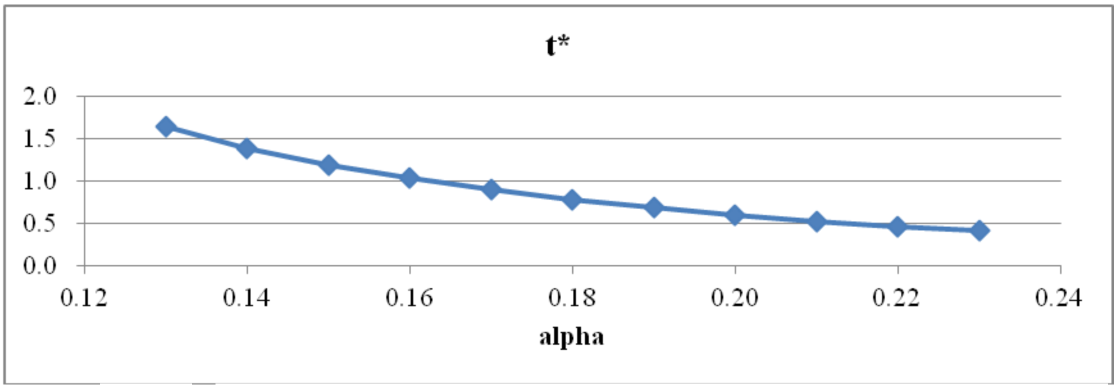

Figure 4.2. The optimal selling timing is changed by housing price growth rate

4.3 The housing return

$$\frac{\partial t^*}{\partial u}=\frac{\left\{\frac{\left[-\frac{\beta_1\cdot(\gamma\cdot I\cdot t_c-b)}{\left[\beta_1(u-\delta)\right]^2}\right]}{\left[I-\frac{b}{\delta}\left(1-e^{-\delta T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1(u-\delta)}\right]}\right\}\cdot(u-\delta)+\ln\left\{\left[I-\frac{b}{\delta}\left(1-e^{-\delta T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1(u-\delta)}\right]\cdot\frac{1}{H_0}\right\}}{(u-\delta)^2} \tag{4.4}$$

We assume housing return is positive because of land development usually arises in booming market. Equation (4.4) and Figure 4.3 show the characteristic for optimal selling timing and numerical results. We can recognize it in the figure with our setting even the relationship is not clear in the function. Numerical results indicate their relationship basically are negative. Moreover, we can find out that higher housing return makes developer prefer to sell pre-sale agreements earlier. Developers would not to hold the right of selling timing as housing return is higher.

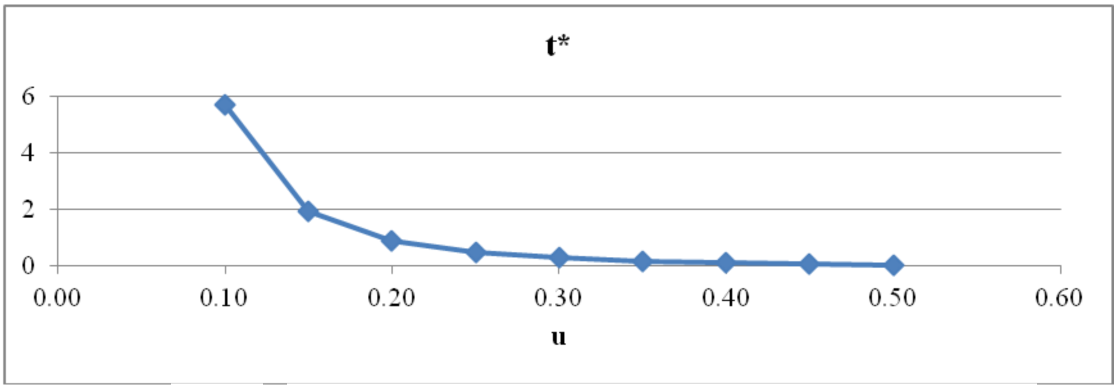

Figure 4.3. The optimal selling timing is changed by housing return

4.4 Housing depreciate rate

$$\frac{\partial t^*}{\partial \delta}=\frac{\left\{\frac{\left[\frac{b-(T+1)\cdot b\cdot e^{-\delta T}}{\delta^2}+\frac{\beta_1\cdot(\gamma\cdot I\cdot t_c-b)}{\left[\beta_1(u-\delta)\right]^2}\right]}{\left[I-\frac{b}{\delta}\left(1-e^{-\delta T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1(u-\delta)}\right]}\right\}\cdot(u-\delta)+\ln\left\{\left[I-\frac{b}{\delta}\left(1-e^{-\delta T}\right)+\frac{\gamma\cdot I\cdot t_c-b}{\beta_1(u-\delta)}\right]\cdot\frac{1}{H_0}\right\}}{(u-\delta)^2} \tag{4.5}$$

Equation (4.5) shows the relationship between housing depreciate rate and optimal developed timing. We still can't make sure their relationship in the function. Figure 4.4 shows the relationship trend by numerical approach. The results indicate positive relation which means developers would housing as complement if housing are highly depreciate rate. Ceteris paribus, the depreciate rate would deduct the housing price growth rate. Housing with higher depreciate rate imply useful life would be shorter. Means that, developer may sell the shorter useful life housing when the construction is near completion.

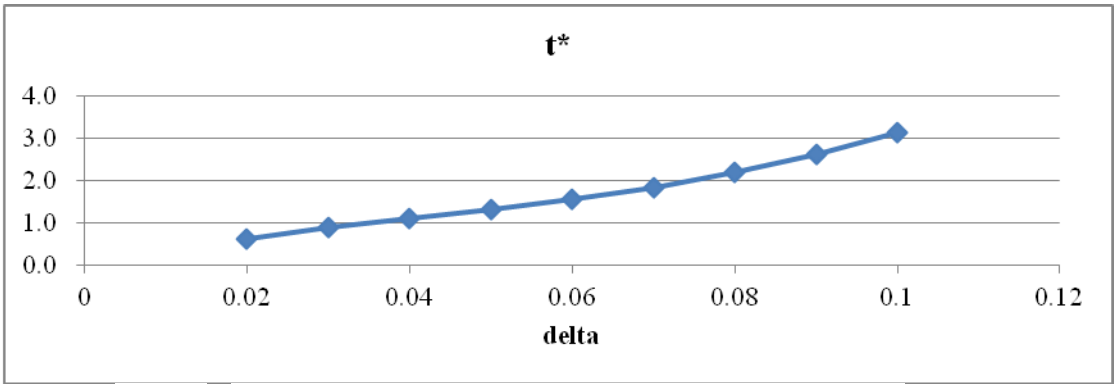

Figure 4.4. The optimal selling timing is changed by housing depreciate rate

4.5 Initial investment

$$\frac{\partial t^*}{\partial I}=\frac{1+\dfrac{\gamma\cdot t_c}{\beta_1(u-\delta)}}{\left[I-\dfrac{b}{\delta}\left(1-e^{-\delta T}\right)+\dfrac{\gamma\cdot I\cdot t_c-b}{\beta_1(u-\delta)}\right]\cdot(u-\delta)} \tag{4.6}$$

To analyze the characteristics of basis process, we derive the sensitivity of the optimal developed timing for initial investment. Base on equation (4.6), we find that the relationship between the optimal developed timing and initial investment depends on many variables. Figure 4.5 shows the numerical result for the relationship with our parameters setting. There is positive trend for t^* as I increase that means the optimal selling timing would be closer to construction complement as initial investment higher. In other words, developers may hold exercise right, make pre-sale agreements, later as initial investment higher.

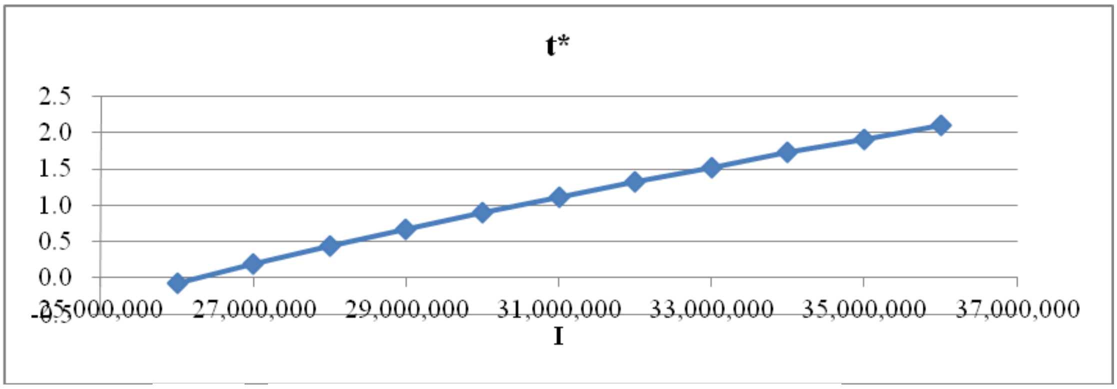

Figure 4.5. The optimal selling timing is changed by initial investment

4.6 The integrated period

$$\frac{\partial t^*}{\partial T}=\frac{-b\cdot e^{-\delta T}}{\left[I-\dfrac{b}{\delta}\left(1-e^{-\delta T}\right)+\dfrac{\gamma\cdot I\cdot t_c-b}{\beta_1(u-\delta)}\right]\cdot(u-\delta)} \tag{4.7}$$

Base on equation (4.7), we derive the relationship between the optimal developed timing and the integrated period. Figure 4.6 shows the negative relationship trend by numerical approach. The negative relationship means the developer would make pre-sale contracts earlier as integrated period longer. A bigger development project may extend the integrated period, developer may reduce uncertainty of the future by make pre-sale agreements earlier.

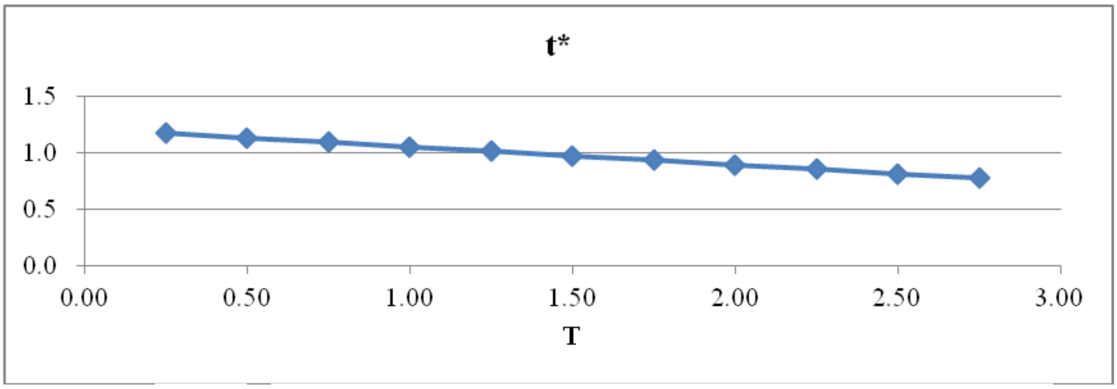

Figure 4.6. The optimal selling timing is changed by integrated period

4.7 Capital structure of developer

$$\frac{\partial t^*}{\partial \gamma} = \frac{I \cdot \text{tc}}{\left[I - \frac{b}{\delta}\left(1 - e^{-\delta T}\right) + \frac{\gamma \cdot I \cdot \text{t}_c - b}{\beta_1 (u - \delta)} \right] \cdot \beta_1} \tag{4.8}$$

The relationship between capital structure of developer and optimal developed timing shows on equation (4.8). Figure 4.7 shows the numerical result and indicates positive relationship trend. The result indicates developer would not prefer to launch pre-sale contracts as their debt ratio increased. The credit level of developer, business cycle, housing market circumstances, etc. all can effect debt ratio of developer. After obtaining funds, developer would hold exercise right longer for predominating more market information.

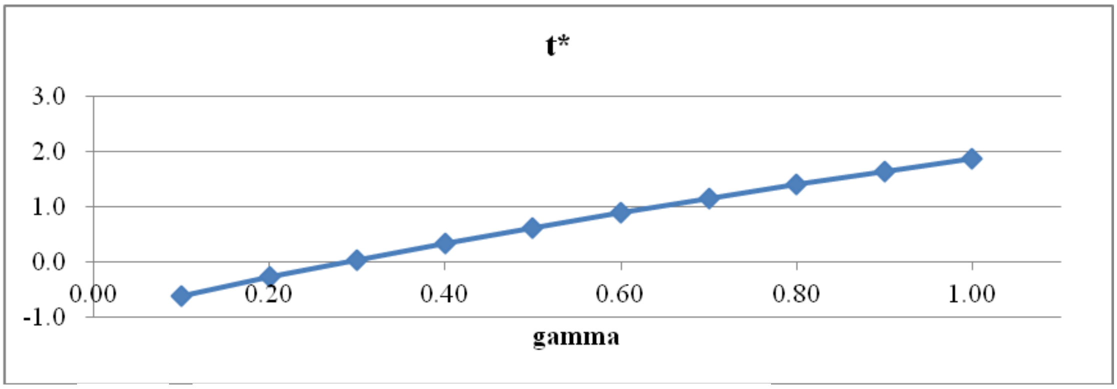

Figure 4.7. The optimal selling timing is changed by capital structure of developer

4.8 Corporate tax rate

$$\frac{\partial t^*}{\partial t_c} = \frac{\gamma \cdot I}{\left[I - \frac{b}{\delta}\left(1 - e^{-\delta T}\right) + \frac{\gamma \cdot I \cdot \text{t}_c - b}{\beta_1 (u - \delta)} \right] \cdot \beta_1} \tag{4.9}$$

Equation (4.9) and Figure 4.8 show the trend for optimal developed timing and numerical results. The relationship is not clear in the function but we can recognize it in the figure. Numerical results indicate their relationship are positive which means developers would not prefer to launch pre-sale contracts as corporate tax rate increased. Developers are more motivated to sell housing later, even completed, because the housing price which is close to completed date collect more market information. It's more equitable to tax for developers.

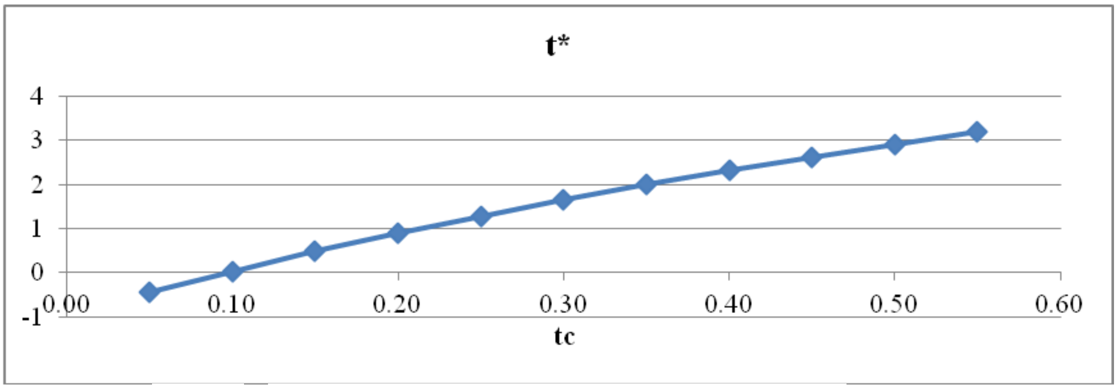

Figure 4.8. The optimal selling timing is changed by corporate tax rate

4.9 Risk-free rate

$$\frac{\partial \beta_1}{\partial r} = -\frac{1}{\sigma^2} + \frac{\left[\frac{(r-\delta)}{\sigma^2} + 0.5\right]}{\sigma^2 \cdot \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - 0.5\right]^2 + \frac{2r}{\sigma^2}}} \tag{4.10}$$

$$\frac{\partial t^*}{\partial \beta_1} = \frac{-(I \cdot \gamma \cdot t_c - b)}{\left[I - \frac{b}{\delta}\left(1 - e^{-\delta T}\right) + \frac{\gamma \cdot I \cdot t_c - b}{\beta_1 (u-\delta)}\right] \cdot \left[\beta_1 (u-\delta)\right]^2} \tag{4.11}$$

Risk-free rate is a function of β_1 as equation (4.10) showing. Consequently, the relationship between risk-free rate and β_1 would effect positive/negative sign between optimal developed timing and β_1. Equation (4.11) can't clearly indicates the relationship between optimal developed timing and β_1. Figure 4.9 shows the numerical result which indicates positive trend for t^* and risk-free rate with our parameters setting. The result shows that developers would prefer to sell housing when it completed as risk-free rate increased.

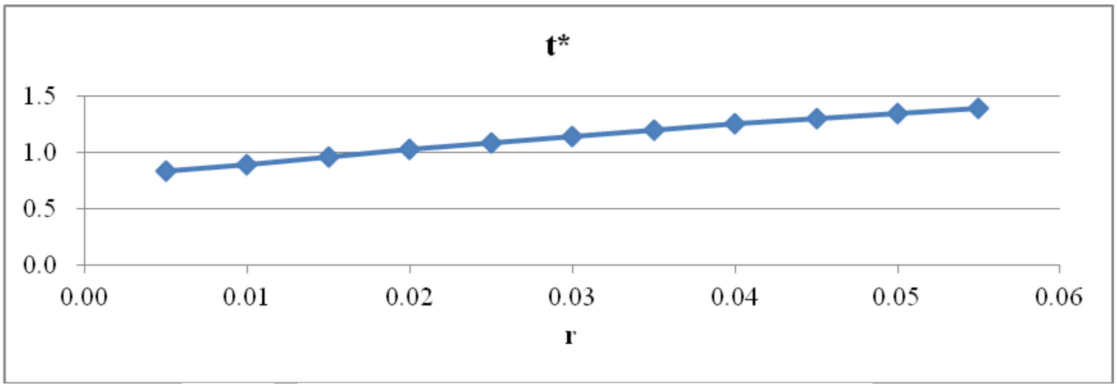

Figure 4.9. The optimal selling timing is changed by risk-free rate

In this paper, we model a general function by real option framework to handle the best selling timing issue. Furthermore, we employ sensitivity analysis to investigate the effective to optimal selling timing from variables. The results indicate that housing price volatility, housing depreciate rate, initial investment, capital structure of developer, corporate tax rate and risk-free rate are positive relationship with optimal selling timing. On the other hand, housing price growth rate, housing return and integrated period are negative relation with optimal selling timing. We can generalize the optimal selling timing strategy for developers by sensitivity analysis. Incidentally, the cost of debt would not effect selling timing decision.

5. Chapter Summary

Selling timing has become one of the most popular issues for housing transactions, maybe pre-sale or completed housing selling. This issue has got more attention in America and Europe areas in recent years. Because of pre-sale contracts, if it is necessary, can spread real estate valuation risks between developers and consumers and among consumers with varied expectations about future real estate prices. Pre-sale not only can help developers as aforementioned, but also stabilize the financial situation.

If developers can wait for the resolution of uncertainty before deciding to pursue the irreversible investment, they can avoid potentially large losses. Real option provides assistance for our consideration on research. Previous studies on real option models apply most directly to individual investment projects in empirical and predict the trigger prices. The other part, empirical studies investigate the effect of uncertainty on the timing of investment, which is a direct test of the optimal exercise of real options. In this paper, we have developed a comprehensive theoretical function, using a real option approach, for analyzing the decision to sell housing under uncertainty while highlighting the role of variable intensity.

We provide an optimal selling timing function for developers by estimating optimal selling housing price on real option framework. It is a comprehensive theoretical function which includes the characteristics of corporation. Numerical approach provides the basis of comparative static analysis with our parameters setting. The numerical results indicate, developers are more motivated to sell housing later, even completed, because the housing price maybe get a higher level with higher housing price volatility, developers may hold exercise right (make pre-sale agreements later) as initial investment higher. The longer integrated period (project period) makes reversed effective for optimal selling timing. Higher housing return (housing price growth rate) makes developer prefer to sell pre-sale agreements earlier. There are different depreciate rate for different locations, construction and planning purposes. The results indicate positive relation which means developers would housing as complement if housing are highly depreciate rate. Developer who have the ability to obtain funding would hold exercise right longer for predominating more market information on corporation sight. Developers also prefer to sell housing when it completed as risk-free rate and corporate tax rate increased. This model also recognizes that developers could optimally presell whenever they are allowed to. In addition, the presale system exists because it allows the developer to share risk (and profit) with buyers (Lai et al., 2004). Sometimes, they want to reserve the value of real option for more market information. We summarize the relationships of determinates with best selling timing (t^*) in Table 5.1.

Table 5.1: The relationships of determinates with t^*

	t^*
u	−
δ	+
σ	+
γ	+
r	+
r_b	×
t_c	+
I	+
T	−

Note: 「+」means positive relationship, 「−」means negative relationship, 「×」means the t^* decision-making will not be changed.

Over the past five decades, pre-sales have become a popular practice for property transactions, particularly in many Asian markets. Most residential condominium sales in Hong Kong, Taiwan, China, Korea, and Singapore. It can clearly be seen, developers care about the selling timing decisions. This research not only provides the optimal selling timing strategy for developers by sensitivity analysis but we can realize the implied meaning for selling timing.

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