GRASP WITH VLSN FOR AN INVENTORY-ROUTING PROBLEM

by

Sombat Sindhuchao
Department of Industrial Engineering,
Ubonratchathani University, Ubonratchathani, Thailand
E-mail: somsin@rocketmail.com

ABSTRACT

This paper presents a solution approach for an inventory routing problem (IRP). An inbound material-collection system is considered. It consists of a set of suppliers who produce non-identical items, and a central warehouse stocking a number of unique items that face constant and deterministic demands from outside retailers. With an economic order quantity (EOQ) inventory policy, the items are jointly replenished and collected by a fleet of identical vehicles that have capacity and frequency constraints and are dispatched from the central warehouse. A greedy randomized adaptive search procedure (GRASP) is proposed to solve the problem. GRASP is an iteration process that consists of two phases, a construction phase and a local search phase. In the construction phase, an initial feasible solution is generated using the greedy heuristic based on the Distance Sum heuristic. In the local search phase, a very large-scale neighborhood search (VLSN) algorithm is applied to improve the solution. To measure the performance of GRASP with VLSN, computational experiments are conducted on randomly generated problems and solutions are compared with the lower bound on the total costs. The results reveal that the GRASP with VLSN performs efficiently in finding near-optimal solutions.

KEYWORDS
Inventory-Routing, Joint Replenishment, Greedy Randomized Adaptive Search Procedure, Very Large-Scale Neighborhood Search

INTRODUCTION

Inventory and transportation are major elements of the logistics system. From Thailand's Logistics Report 2011 (2012), the inventory cost and transportation cost accounted for 91.2% of the total logistics cost. Usually, there is a trade-off between both costs. As a result, inventory control and transportation planning should be closely coordinated to achieve the best trade-off.

In this paper, an inventory routing problem (IRP) for an inbound commodity collection system is studied. The system comprises a central warehouse with unlimited stocking area and a set of suppliers each of whom produces one or more but non-identical items. Each item faces a constant and deterministic demand rate from outside retailers. An economic order quantity (EOQ) policy is adopted for joint replenishment of a group of items. No item can be assigned to more than one group. When inventory replenishment is needed, a fleet of identical vehicles with limited capacity are sent from the warehouse to collect groups of items. In addition to the vehicle capacity constraint, each vehicle cannot travel more than a specific number of trips per year. A Greedy Randomized Adaptive Search procedure (GRASP) [see Resende (1998)] is developed to solve this IRP. In the construction phase, a greedy heuristic based on the Distance Sum heuristic [Sindhuchao (2006)] is implemented to obtain an initial solution. A very large scale neighborhood (VLSN) search is applied as a local search in the local search phase of GRASP.

A number of related researches involving coordination of inventory and vehicle routing were done in the past. For a single item case, Federgruen and Zipkin (1984) study the allocation of a scarce resource from a central depot to retailers using a fleet of capacitated vehicles and random demands are considered in a single period model. They formulate the problem as a non-linear integer program and modify interchange heuristics to solve the deterministic vehicle routing problem. Anily and Federgruen (1990) study a similar case where the demand rate of each retailer is an integer multiple of some base rate. A two-echelon distribution system is developed by Anily and Federgruen (1993). They include inventory kept at retailers as well as at a central warehouse. Anily (1994) extend their work by assuming general holding cost rates. Bertazzi et al. (2002) adopt an order-up-to level policy and propose a heuristic to solve IRP. Then, Cousineau-Ouimet resolves their problem using a Tabu Search approach. Zhong and Aghezzaf (2008) focus on the long-term single-vehicle IRP with constant demand rates. They solve the problem by developing a hybrid approximation approach. The most recent work about IRP with a single product belongs to Aghezzaf et al. (2012). They formulate the problem as a mixed-integer program and develop an exact method as well as a saving-based heuristic. Comparison of results from both approaches shows that the saving-based heuristic does not work well.

Not so many papers study IRP with multiple items. Among them are Viswanathan and Mathur (1997), Qu et al. (1999) and Sindhuchao et al. (2005). Viswanathan and Mathur (1997) propose a stationary nested joint replenishment policy for IRP in a single warehouse multi-retailer multi-item distribution system with deterministic demands. Under this policy, replenishment intervals must be power of two multiples of a base planning period. Qu et al. (1999) investigate the uncertain demand rate case with unlimited vehicle capacity. They adopt a policy where each joint replenishment interval is an integer multiple of a base interval and propose a decomposition method to solve the problem. Sindhuchao et al. (2005) study the same problem as the one of this paper. They develop a Branch and Price algorithm to solve for an optimal solution. They also propose heuristics with VLSN as a local search that can solve a large problem size efficiently. For more review of the literature of IRP, see the work of Andersson et al. (2010).

The rest of this paper is organized as follows. The description of the problem and the model formulation are discussed in section 2. In section 3, GRASP with VLSN is developed. In section 4, computational tests are conducted and the research is concluded in section 5.

PROBLEM DESCRIPTION AND MODEL FORMULATION

An integrated inventory and transportation system consists of a central warehouse stocking a set of items, S, and a set of suppliers who produce one or more non-identical items. The demand rate D_j for item j and $j \in$ S is assumed to be constant and deterministic. To jointly replenish inventory, the items are collected from suppliers by a fleet of m identical vehicles with limited capacity C. Each vehicle cannot travel more than a certain number of trips per year F. This means that the total quantities picked up by each vehicle per year cannot exceed CF. An economic order quantity (EOQ) policy is chosen for jointly replenishing a subset of items S. No item can be assigned to more than one subset. Each subset of items is replenished with a single vehicle. The total system costs comprise the inventory holding cost associated with each item and incurred at a constant rate of h_j per unit item j per year, the fixed ordering cost, the fixed dispatching cost and the vehicle routing cost. For convenience, the fixed ordering cost and the fixed dispatching cost are combined in a single term K. The inventory routing problem is to determine the subsets of items, the corresponding replenishment quantities, the replenishment interval and the optimal vehicle routes, that minimize the average total inventory and transportation cost per unit time.

For convenience, it is assumed that the units of items can be meaningfully added together. Let $S \subseteq$ S, and let Q_j denote the replenishment quantity of item j. Then define $Q(S) = \sum_{j \in S} Q_j$ and $D(S) = \sum_{j \in S} D_j$ to be the aggregate replenishment quantity and the aggregate demand for subset S respectively. The weighted average inventory holding costs with respect to the aggregate replenishment quantity can be defined as $h(S) = (\sum_{j \in S} h_j D_j) / D(S)$.

For a subset of items S, the fixed ordering cost, the fixed dispatching cost and the vehicle routing cost are constant for each replenishment. As a result they can be combined as a single term denoted by $L(S)$. With both the vehicle capacity and frequency constraints, the aggregate replenishment quantity for the items in a feasible subset S can be determined from equation (1)

$$Q^*(S) = \max\left\{\frac{D(S)}{F}, \min\left\{\sqrt{\frac{2D(S)L(S)}{h(S)}}, C\right\}\right\} \tag{1}$$

and the corresponding optimal costs can be obtained from equation (2).

$$c(S)=\begin{cases} L(S)F+\frac{1}{2}h(S)\frac{D(S)}{F} & \text{if } \sqrt{\frac{2D(S)L(S)}{h(S)}}\le\frac{D(S)}{F} \\ \sqrt{2D(S)L(S)h(S)} & \text{if } \frac{D(S)}{F}\le\sqrt{\frac{2D(S)L(S)}{h(S)}}\le C \\ L(S)\frac{D(S)}{C}+\frac{1}{2}h(S)C & \text{if } C\le\sqrt{\frac{2D(S)L(S)}{h(S)}} \end{cases} \quad (2)$$

After the aggregate replenishment quantity for the items in a feasible subset S is determined, the optimal replenishment interval of this subset $T^*(S)$ and the optimal replenishment quantity of each item j, Q_j^*, in this subset can be computed from equation (3) and (4) respectively.

$$T^*(S)=\frac{Q^*(S)}{D(S)} \quad (3)$$

$$Q_j^*=D_jT^*(S)=D_j\frac{Q^*(S)}{D(S)} \quad (4)$$

Suppose there are m vehicles available. So, the integrated inventory routing problem can be formulated as a partitioning problem (5)-(7).

$$\min\sum_{i=1}^{m}c(S^{(i)}) \quad (5)$$

subject to

$$\bigcup_{i=1}^{m}S^{(i)}=\mathrm{S} \quad (6)$$

$$S^{(i)}\cap S^{(k)}=\phi \qquad \text{for all } i,k=1,2,...,m;\quad i\ne k. \quad (7)$$

$S^{(i)}$ denotes a subset of items collected by vehicle i. Constraints (6) and (7) define that each item must be assigned to only one subset. In other words, an item is collected by a single vehicle and there is no split order for each item. See Sindhuchao et al. (2005) for more detail of model formulation.

GRASP WITH VLSN

A greedy randomized adaptive search procedure (GRASP) is an iteration process. It consists of two phases, a construction phase and a local search phase. In the construction phase, an initial feasible solution is generated. Usually in this phase, a greedy heuristic is applied to construct a solution by randomly adding one element at a time to the solution. In the local search phase, the solution is improved using a local search method. The process is repeated for a predetermined number of times M and the best overall solution is considered as the solution of the problem obtained from GRASP. The fundamental of GRASP is that solving the problem with multiple starting solutions has more chances to achieve a near optimal solution. Consequently the performance of GRASP mainly depends on the efficiency of the local search method. The larger the neighborhood search, the better the solution.

The application of GRASP has been seen on various problems. For IRP, Sindhuchao et al. (2004) develop a Distance Sum (DS) heuristic and execute the heuristic in the construction phase. They employ a local search method called One Supplier Move (OSM) to improve the solution in the local search phase. Shen et al. (2008) formulate a mixed-integer nonlinear programming model of IRP in crude oil transportation with multiple transportation modes and propose to find near optimal solution. Dubedout et al. (2012) study a gas distribution system and propose two different GRASP approach: single start and multi start GRASP. Various local search methods are utilized. The computational results show that the multi start GRASP outperforms the single start GRASP.

In this paper, GRASP with VLSN is developed. In the construction phase, the DS based heuristic is implemented to separate items in different groups and assign each group to each vehicle. The vehicle routes are constructed using the insertion heuristic. In the original DS heuristic, the best item with smallest distance-sum is selected to construct a solution in each iteration. However, in the DS based heuristic, an item that will be added to build a solution is randomly chosen from the Restricted Candidate List (RCL). For this paper, the unassigned items whose distance-sum is greater than the best distance-sum at most a certain value α will be included with the best one in the RCL. In the local search phase, I-VLSN [see Sindhuchao et al. (2005)] is employed to improve the initial solution. In I-VLSN, to obtain a better solution, items are considered as elements to move from one group to others. VLSN algorithms have very large neighborhoods to search. To reduce computational time of searching for the improved solution, an improved neighbor must be identified quickly without explicit enumeration and evaluation of all neighbors in the neighborhood. Ahuja et al. (2000) develop an efficient method for identifying an improved neighbor based on a characterization of the neighborhood through an improvement graph [see Ahuja et al. (2002)]. For application of VLSN, see Sindhuchao (2006) and Geng et al. (2005).

A greedy heuristic that can be used to generate an initial feasible solution and a neighborhood search algorithm called One Supplier Move (OSM) are proposed. In addition, a greedy randomized adaptive search procedure (GRASP) is also described.

Distance Sum (DS) Heuristic

The DS heuristic constructs routes sequentially for one vehicle at a time. An item is added to a vehicle, one item at a time, based on the idea that the best candidate should be an item whose supplier is located closest to the warehouse and also close to at least one supplier that is already in the route. Let $d_{j_1 j_2}$ denote the distance (cost) from the supplier of item j_1 to the supplier of item j_2, for all j_1, j_2 $\in$ S. Similarly, let d_{0j} and d_{j0} denote the distance from the warehouse to the supplier of item j.

Step0. Initialize an empty route for the next vehicle.

Step1. For each of ungrouped items that can be added to the vehicle without violating its capacity constraint, say j, determine its distance-sum as the minimum value of $d_{jj'} + d_{0j}$ over all items j' served by the current vehicle. If the current vehicle does not contain any items, let the distance-sum be d_{0j}. If no such items exist, go to Step 3.

Step2. Find the item with the smallest distance-sum, assign it to the vehicle, and return to Step 1.

Step3. If all items have been assigned to a vehicle, go to Step 4. Otherwise, if there are available vehicles left, return to Step 0.

Step4. Find a TSP tour for all vehicles using the Arbitrary Insertion heuristic to construct a route and the 2-opt exchange heuristic to improve the tour.

VLSN

The general procedure of the I-VLSN can be described as follows.

Step 1. Generate an initial feasible solution using one of proposed heuristics.

Step 2. Construct the improvement graph for the cyclic exchange and path exchange neighborhoods. See Figures 1 and 2.

Step 3. Identify a cost-decreasing cyclic exchange or path exchange. If one of them exists, improve the solution by moving items according to the cost-decreasing exchange identified and return to step 2. Otherwise, stop.

FIGURE 1
CYCLIC EXCHANGE

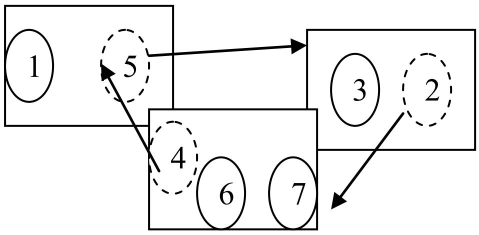
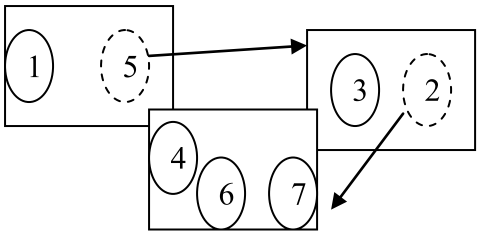


FIGURE 2
PATH EXCHANGE



GRASP with VLSN

The procedures of GRASP with VLSN can be explained below.

Step 1. Set k=1, a number of iteration M and α .
Construction phase
Step 2. Generate an initial solution using the DS based heuristic.
Local search phase
Step 3. Improve the solution obtained from Step 2 using I-VLSN.
Step 4. Update the best solution found and set k=k+1. If k is less than M, return to Step 2. Otherwise, stop.

COMPUTATIONAL EXPERIMENTS

For computational tests, random instances from Sindhuchao et al. (2005) have been used. The problem details are as follows. The demand rate and the inventory holding cost rate for each item are randomly generated from the uniform distribution on [100,300] and [1,15] respectively. The items are randomly assigned to one of ten suppliers. The warehouse's and suppliers' locations are generated uniformly in the square $[0,20]^2 \subset \mathrm{R}^2$, and Euclidean distances are used to measure transportation costs, with unit cost per unit distance traveled. A base case has been defined as follows: A vehicle capacity of C is 150 units, the fixed costs K are set to 50, and the maximum number of trips allowed per time unit by each vehicle F is 10. For the GRASP with VLSN, the maximum number of runs M is set to 50 and α is set to 50% for the DS based heuristic. The size of an instance is identified by the number of items, n , and the number of vehicles, m . Four problem sizes are tested with ten instances each. The GRASP with VLSN has been implemented in the C++ programming language on a laptop with a 1.99 GHz Intel Core Duo CPU and 0.99 GB of RAM.

To measure the performance of the GRASP with VLSN, the solutions are compared to the lower bound (LB) computed by a column generation approach in the work of Sindhuchao et al. (2005) and the optimal solution for the small problem size (n=15,m=3). The results are shown in Tables 1-3

TABLE 1
TOTAL COSTS COMPARISON BETWEEN THE LOWER BOUND, THE OPTIMAL SOLUTION AND THE GRASP-VLSN

Problem No.	n=15,m=3	n=30,m=6	n=40,m=8	n=50,m=10
1	0.00	1.67	1.50	2.68
2	1.30	2.00	3.33	1.75
3	2.10	3.14	2.98	1.94
4	3.43	2.79	2.53	2.74
5	6.69	3.08	2.25	2.40
6	5.48	1.90	1.88	1.45
7	0.96	2.93	1.75	1.88
8	2.60	2.72	2.06	2.24
9	1.00	3.44	1.90	1.91
10	4.89	2.34	2.20	2.64
Maximum	6.69	3.44	3.33	2.74
Average	2.85	2.60	2.24	2.16

TABLE 2
PERCENT DEVIATION FROM THE LOWER BOUND FOR SOLUTIONS OBTAINED FROM THE GRASP WITH VLSN

Problem No.	LB	Optimal	GRASP-VLSN	% error from LB	% error from optimal
1	2778.1	2778.1	2778.1	0.0	0.00
2	2645.8	2645.8	2680.2	1.3	1.30
3	2545.2	2598.6	2598.6	2.1	0.00
4	2669.7	2761.2	2761.2	3.4	0.00
5	2557.6	2726	2728.8	6.7	0.10
6	2563.9	2699.3	2704.4	5.5	0.19
7	2511.3	2526.7	2535.5	1.0	0.35
8	2378.3	2426.2	2440.2	2.6	0.58
9	2556.2	2577.2	2581.7	1.0	0.18
10	2710.0	2825.5	2842.5	4.9	0.60
Maximum	2778.1	2825.5	2842.5	6.7	1.3
Average	2591.6	2656.5	2665.1	2.8	0.3

TABLE 3
AVERAGE COMPUTATIONAL TIME FOR THE OPTIMAL SOLUTION, LOWER BOUND (LB) AND GRASP WITH VLSN

Problem Size	Computational Time (second)		
	Optimal	LB	GRASP-VLSN
n=15,m=3	13571.3	13.5	0.0921
n=30,m=6	N/A	849.5	0.8907
n=40,m=8	N/A	14207.6	2.1406
n=50,m=10	N/A	48672.0	4.8143

From the computational results in Table 1 and 2, the GRASP with VLSN perform quite well with largest deviation of 6.69% from the lower bound and the smallest average percent deviation is just 2.16%. Compared with the optimal solution for n=15 and m=3, the average gap is only 0.3%. It is also found that the performance of GRASP with VLSN improves as the problem size increases. For the problem size of n=50 and m=10, the error bounds of the solutions obtained from the GRASP with VLSN are less than 3% in all instances. In addition, the GRASP with VLSN can find a near optimal solution in less than 5 seconds.

CONCLUSION AND FUTURE RESEARCH

In this paper, an inbound material-collection system with one warehouse, multiple supplies and multiple items is studied. A greedy randomized adaptive search procedure with VLSN for an integrated inventory-routing problem in a deterministic setting is developed. The solutions obtained from the GRASP with VLSN are compared with the lower bound obtained from a column generation approach. The computational results indicate that the GRASP with VLSN can find near-optimal solutions quickly with an average error bound less than 3% for most cases. For future research, a more efficient constructive heuristic will be developed to reduce the percent deviation from LB. Moreover, to get a strong support to the results of this paper, GRASP with VLSN will be implemented to solve larger problem sizes of IRP and also various types of problems.

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